Brane Cosmology with the Chameleon Scalar Field in Bulk

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(Dated: February 25, 2012)

In this work we investigate the effect of a kind of scalar field, called chameleon, on the evolution of universe. We put this scalar field in the bulk. It is displayed that this scalar field gives us an exponential expansion in early time which may concern inflation. Interaction between scalar field and matter bring some complications in our analysis, however it is shown that by defining an effective potential, we could recover conventional equation at inflation era. After inflation, and entering the universe in radiation era, the exponential expansion is omitted. Also, in late time the universe possess an accelerated expansion. In the last section, the validity of generalized second law of thermodynamic, with assumption the validity of first law is considered.

PACS numbers:

I. INTRODUCTIONS

In the last decades, an increasing number of data, such as astrophysical data from type Ia supernovae [1], and cosmic microwave background radiation (CMBR), indicate that our universe is undergoing an accelerated expansion. Since the ordinary matter cannot cause this expansion, there are introduced another type of matter, called dark energy with a negative pressure, responsible for this accelerated expansion. Dark energy is one of the most puzzling aspects of our observed universe. It seems that the best and simple candidate for dark energy is cosmological constant, with equation of state $p = -\rho$, but it has some problem like "fine tuning". Scalar fields are introduced as another model for explanation dark energy. They treat scalar field as dark energy component with a dynamical equation of state. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggest that energy forms with negative pressure is provided by a scalar field evolving down a proper potential. So far, people have investigated a large class of scalar field models. A most general model for cosmic acceleration is a slow-rolling scalar field, called quintessence [2]. The slow roll means that the scalar field has negative pressure and then causes to positive accelerating expansion. Other attempt to explain dark energy is phantom field [3], as well as modification gravitational theory [4]. The equation of state of phantom model is displayed by $p = \omega\rho$ where ω is smaller than -1. Another interesting scalar field model is named "Chameleon". This scalar field has been suggested by Khoury and Weltman [5]. In that model a coupling to the matter is proposed which gives the scalar field a mass depending on the local den-

sity of matter (for a useful ref. See [6]), a specific example of a chameleon field arising from string theory is given in [7]. This dependence on the local density of matter cause chameleon field has a suitable value of mass so can have a good result in the solar system, whereas quintessence is not an appropriate model in this scale because the value of mass of scalar field is small. It is one of advantage of chameleon scalar field.

Another model which has attracted a huge attention, is the theory of extra-dimension where all kind of matter and their interaction are confined on a hypersurface (brane), except gravity which can propagate along the fifth dimension, embedded in a higher-dimension space-time (bulk). Since dark energy and dark matter is detected only by its gravitational interaction, it may interpret as the gravitational effects of other branes in the bulk. Because of this and some other attractor properties, this model have received much attention recently. The possibility that our four-dimensional universe may be embedded in a higher-dimension bulk space-time is motivated by superstring theory and M-theory. Higher-dimension models have a long history, but it revived by works of L. Randall and R. Sundrum in 1999 [8]. They have introduced two models in order to solve the hierarchy problem in particle physics, however after a while these two models, because of their interesting properties, could attract a salient attention in cosmology. In their first model they suppose two brane which our brane has a negative tension (T. Shiromizu *et al* [10] have shown that this model is un physical). In their second model they suppose a brane with infinite extra dimension. In that model our universe has a positive tension. The effective four-dimensional gravity in the brane is modified by extra dimension [10, 11]. There is some correction in generalized Friedmann equation like its dependence on quadratic brane energy density that couples directly to the five-dimensional Planck scale. The classical Friedmann equation can be recovered in the low energy in

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late time, when energy density is much smaller than the brane tension. In Randall-Sundrum brane world scenario the bulk contains only negative cosmological constant. This needs a fine-tuning between cosmological constant of bulk with tension of brane. It may be more desirable to introduce a model without requirement fine tuning. String/M-theory suggests that it will also contain scalar field which are free to propagate through the bulk [13], so it is natural and attractor to consider the existence of a scalar field in the bulk. This has been investigated in several works such as [14, 15]. The existence of bulk matter or scalar field can influence the cosmological evolution on the brane. The interesting properties of these two models, namely chameleon scalar field and brane world scenario, have motivated us to investigate the effects of a chameleon scalar field in bulk on the evolution on universe.

The plan of this paper is as following: In section two, the action of brane and bulk is introduced. This action is similar to [5] , as one can see scalar field is embedded in bulk space time and this scalar field can interact with matter which is confined on the brane. In addition some note related to gravity localization and basic equations is given. With the help of five-dimensional action and supposing metric fluctuation, the wave equation for transverse-traceless mode of the metric fluctuation is obtained. The wave equation help us to determine whether the fluctuation mode fall or rise away from the brane, and thus whether or not gravity actually localized the brane. In next part, the junction conditions, basic evolution equations, and scalar field equation of motion is deriven,, and it is shown that the matter density because of interaction with matter is not conserved. In section three, the basic evolution equation on the brane (our universe) are acquired. With investigation the evolution of universe we realize that there is an exponential accelerate expansion is early times, so it is tried to have some explanation related to inflation and reheating, and they are considered in some more detail. By passing time and entering the universe in radiation dominant era, exponential term is omitted. In late time, the universe possess a positive accelerated expansion. In section four, we consider the validity of generalized second law of thermodynamic (GSLT) of this model in an accelerated expanding universe for apparent horizon and cosmological event horizon. It is indicated that when matter on the brane is a sort of phantom or quintessence, the validity of GSLT is completely confirmed.

II. GENERAL FRAMEWORK

To begin our work, we consider the following action

$$
S = \int d^5 x \sqrt{-g} \Big\{ \frac{M_5^3}{2} R^{(5)} - \frac{1}{2}(\nabla\phi)^2 - V(\phi)) \Big\} \\
- \int d^4 x \mathcal{L}_m(\psi_m, \tilde{h}_{\mu\nu}), \tag{1}
$$

the first term describes the five-dimensional gravity in the presence of a scalar field ϕ and the second term describes the matter of brane that are coupled to scalar field by

$$
\tilde{h}_{\mu\nu} = \exp\big(\frac{2\beta\phi}{M_p^{3/2}}\big)h_{\mu\nu}, \tag{2}
$$

where β is a dimensionless coupled constant, and M_p is the effective four-dimensional Planck mass on the brane. Make attention that the brane tension is hidden on the last term action, namely $\mathcal{L}_m$, and tension appears on the brane energy density in evolution equation (this feature can be seen in some paper such as [11, 12]). Note that $\mathcal{L}_m$ is a psedudo-scalar density of weight 1. In Eq.(1), $g_{\mu\nu}$ is five dimensional metric, with signature $(-,+,+,+,+)$, and $h_{\mu\nu}$ is induced metric of $g_{\mu\nu}$, and denotes four dimensional metric of brane. Matter minimally couples to $\tilde{h}_{\mu\nu}$ related to the induced metric $h_{\mu\nu}$ by a conformal transformation. One can obtain the scalar field equation by varying the motion with respect to ϕ as

$$
\nabla^2\phi = V_{,\phi}(\phi) + \frac{1}{\sqrt{-h}}\frac{2\beta}{M_p^{3/2}}\frac{\partial\mathcal{L}_m}{\partial\tilde{h}_{\mu\nu}}\tilde{h}_{\mu\nu}\delta(y), \tag{3}
$$

where $V(\phi)$ is a potential of the chameleon scalar field, which is almost flat. More information about the potential and its feature can be found in [5, 6]. Varying with respect to the metric $g_{\mu\nu}$, the Einstein equation are obtained as

$$
{}^{(5)}G^{\mu\nu} = \kappa_5^2 \Big\{ T^{(\phi)\mu\nu} + T^{(b)\mu\nu} \Big\}, \tag{4}
$$

where $T^{(\phi)\mu\nu}$ stands for the scalar field of total energy-momentum tensor

$$
T^{(\phi)\mu\nu} = \nabla^\mu\phi\nabla^\nu\phi - g^{\mu\nu}(\frac{1}{2}(\nabla\phi)^2 + V(\phi)),
$$

with attention to the dimension of the component of energy-momentum tensor, we realize that the dimension of the scalar field is no longer M, rather it is $M^{\frac{3}{2}}$. Due to this fact, we select $M_p^{\frac{3}{2}}$ in the exponential of Eq.(2), to make it dimensionless. Also the dimension of potential is M^5. Since we put scalar field in five-dimensional space-time, all of these results are acquired. $T^{(b)\mu\nu}$ stands for brane energy momentum tensor part of total energy-momentum tensor, which is as

$$
T^{(b)\mu\nu} = \frac{2}{\sqrt{-h}}\frac{\partial\mathcal{L}_m}{\partial h_{\mu\nu}}\delta(y),
$$

where it can be rewritten as

$$
T^{(b)\mu}_\nu = \frac{\delta(y)}{b}diag(-\rho_b, p_b, p_b, p_b, 0).
$$

Assuming that matter do not interact with each other, so the energy-momentum tensor is conserved in A-frame [6], namely,

$$\tilde{D}_\mu \tilde{T}^{(b)\mu\nu} = 0, \qquad (5)$$

where $\tilde{D}$ indicates covariant derivative in four-dimensions space-time, and $\tilde{T}^{(b)\mu\nu}$ is described by

$$\tilde{T}^{(b)\mu\nu} = \frac{2}{\sqrt{-\tilde{h}}} \frac{\partial \mathcal{L}_m}{\partial \tilde{h}_{\mu\nu}} \delta(y). \qquad (6)$$

This equation gives us the conservation relation in A-frame,

$$\dot{\tilde{\rho}} + 3\tilde{H}(\tilde{\rho} + \tilde{p}) = 0.$$

We suppose that the fluid in brane be a perfect fluid with $\tilde{p} = \omega\tilde{\rho}$, so

$$\tilde{\rho} = \tilde{\rho}_0 \tilde{a}^{-3(1+\omega)} \qquad (7)$$

where $\tilde{\rho}_0$ is a constant. In above equations y displays the coordinate of fifth dimension and $\delta(y)$ explains that our matter is confined to a four dimensional hypersurface, namely our universe.

A.   Gravity Localization on Branes

Although this paper is focused on evolution of universe by assuming a scalar field on bulk which is coupled with matter, we will motivate a class of localized graviton on the brane. In fact as we will show below, this motivation can be partly justified. The five-dimensional action of the model in the bulk (1) can be express as

$$S = \int d^5x \sqrt{-g} \Big(\frac{M_5^3}{2} R - \frac{1}{2} g^{MN} \partial_M \phi \partial_N \phi - V(\phi) \Big) \quad (8)$$

which there is a scalar field ϕ with a potential $V(\phi)$ and the five-dimensional Ricci scalar, ^5R. Here the Latin index run from 0...4 and the Greek index run from 0...3. The space-time metric is supposed as

$$ds^2 = g_{MN} dx^M dx^N \qquad (9)$$

and one can redefine the above metric as following

$$ds^2 = e^{2A(y)} h_{\mu\nu}(x) dx^\mu dx^\nu + dy^2 \qquad (10)$$

where y stands for extra dimension and $e^{2A(y)}$ is the warp factor. Here $h_{\mu\nu}$ is four dimensional metric. We can have following definition for metric (23)

$$ds^2 = e^{2A(z)} \Big(h_{\mu\nu}(x) dx^\mu dx^\nu + dz^2 \Big) \qquad (11)$$

which a coordinate transformation has been imposed as $dz = e^{-A(y)} dy$. In order to study the localized gravity on the 3-brane, one should consider the equation of motion

for linearized metric fluctuation (11). Let us consider the metric fluctuation $\delta g_{MN} = e^{2A(z)} h_{MN}$. So the metric is rearranged as

$$ds^2 = e^{2A(z)} \Big((\hat{h}_{\mu\nu}(x) + \bar{h}_{\mu\nu}(x,z)) dx^\mu dx^\nu + dz^2 \Big) \quad (12)$$

which the axial gauge $\bar{h}_{5M} = 0$ has been imposed on metric. In general, because gravity is coupled to scalar field, fluctuation of scalar field should be considered as the same time when we study fluctuation of metric background. However, following [30], we can only investigate transverse-traceless (TT) modes of the metric fluctuation, namely $\bar{h}_{\mu\nu}^{TT}$. In fact the scalar fluctuation vanished under TT gauges. Therefore the dynamics equation of transverse-traceless modes of the metric fluctuation can be written as [30]

$$\Big(\partial_z^2 - 3(\partial_z A)\partial_z - \hat{g}^{\alpha\beta} \nabla_\alpha \nabla_\beta \Big) \bar{h}_{\mu\nu}^{TT}(x,z) = 0. \qquad (13)$$

The equation could have a solution which is given by $\bar{h}_{\mu\nu}^{TT}(x,z) = K_{\mu\nu} e^{ipx}$ (where $K_{\mu\nu}$ is a constant tensor and $p^2 = -m^2$). Finally by using KK decomposition, the schrodinger equation can be achieved as

$$\Big(-\partial_z^2 A + V(z) \Big) \psi(z) = m^2 \psi(z) \qquad (14)$$

where the localizing potential is defined as

$$V(z) = \frac{3}{2} \Big[\partial_z^2 A + \frac{3}{2} (\partial_z A)^2 \Big] \qquad (15)$$

for more detail refer to [30]. According to [31], it is realized that for trapping the massless mode of gravity, the potential $V(z)$ should have a well with a negative minimum inside the brane and satisfy $V(z) > 0$ far from the brane, namely for $z \to \pm\infty$. By setting $m = 0$, the zero mode wave function can be expressed as $\Psi_0(z) = A_0 \exp(3A(z)/2)$, where A_0 is a constant. In order to localize the four dimensional gravitation, $\Psi_0(z)$ has to obey the normalization constraint

$$\int \|\Psi_0(z)\|^2 dz = c,$$

where c is a finite constant. It is well known that the character of graviton localization depends on the potential V_{QM} and also depends on the warp factor. In braneworld scenario, the four-dimensional effective action is obtained from the five-dimensional action as

$$S \sim M_5^3 \int d^5x \sqrt{-g} R_5 \sim M_p^2 \int d^4x \sqrt{-h} \hat{R}_4 \qquad (16)$$

where M_p is the four-dimensional Planck scale. The localized zero mode will cause a four-dimensional Newtonian interaction potential. In [28] a dS thick brane type of action (9) is investigated and they have obtained the gravitational potential between two point-like mass on brane. They fund that the effective potential between

two point-like mass is from the contribution of the zero mode and the continuum KK modes, and is expressed as [28]

$$U(r) = G_N \frac{M_1 M_2}{r} + \frac{M_1 M_2}{M_5} \int dm \frac{e^{-mr}}{r} \mid \psi_m(0) \mid^2 \quad (17)$$

where the first term is standard Newtonian potential related to the contribution of zero mode, and the second term is the correction of Newtonian potential related to the contribution of KK modes.

This work is focused on evolution of universe by assuming a scalar field on bulk which is coupled with matter. Investigation of gravity localization in detail in the model needs more offers and studying.

B.   Equation of Motions on the Brane

Since we are interesting to study the positive accelerating expansion of the universe, we introduce a special case of metric instead of (9). So we continue our work with a FLRW metric as

$$ds^2 = -n^2(t,y)dt^2 + a^2(t,y)\gamma_{ij}dx^i dx^j + b^2(t,y)dy^2, \quad (18)$$

with a maximally symmetric 3-geometry γ_{ij}. We assume that the brane is embedded on $y = 0$, also we take account Z_2-symmetry. It should be mentioned that the metric is continuous but their first derivative with respect to y is discontinuous, and their second derivative with respect to y includes the Dirac delta function. Substituting the above metric, one can obtain the non-vanishing component of Einstein tensor in the following form

$$^{(5)}G_{00} = -\frac{3}{a^2}\Big\{ -\dot{a}^2 + a'^2 n^2 + aa''n^2 \Big\}, \quad (19)$$

$$^{(5)}G_{ij} = \frac{1}{n^3}\Big\{ 2a\ddot{a}n + 2\dot{n}a\dot{a} + 2n^2 n' aa'$$
$$-n\dot{a}^2 + a'^2 n^3 + 2aa''n^3 + n^2 a^2 n'' \Big\}\delta_{ij}, (20)$$

$$^{(5)}G_{05} = -\frac{3}{an}\Big\{ \dot{a}'n - n'\dot{a} \Big\}, \quad (21)$$

$$^{(5)}G_{55} = \frac{3}{a^2 n^3}\Big\{ -a\ddot{a}n + \dot{n}a\dot{a} + n^2 n' aa' - n\dot{a}^2$$
$$+a'^2 n^3 \Big\}. \quad (22)$$

Note that in the above equations we take $b(t,y) = 1$, and dot denotes derivative with respect to time and prime denotes derivative with respect to fifth coordinate. Since, the second derivative of metric consists the Dirac delta function, according to [11] one can define it as

$$a'' = \hat{a}'' + [a']\delta(y),$$

where $\hat{a}''$ is the non-distributional part of the double derivative of $a(t,y)$, and $[a']$ is the jump in the first derivative across $y = 0$, which defined by

$$[a'] = a'(0^+) - a'(0^-).$$

The junction functions can be obtained by matching the Dirac delta function in the component of Einstein tensor with the component of brane energy-momentum tensor. From the $(0,0)$ and (i,j) component of field equation we have, respectively

$$\frac{[a']}{a_0} = -\frac{\kappa_5^2}{3}\rho_b, \quad (23)$$

$$\frac{[n']}{n_0} = \frac{\kappa_5^2}{3}(2\rho_b + 3p_b), \quad (24)$$

(note that, here, ρ_b and p_b, the brane energy density and pressure respectively, include tension of brane, namely $\rho_b = \rho + \sigma$ and $p_b = p - \sigma$, where ρ and p are matter density and pressure respectively; see [11, 12]). These equations are as same the junction relations which *P. Binetruy et. al.*[11] have obtained in their paper. However, we should make attention here that the energy density and pressure, namely ρ_b and p_b, depend on scalar field. We shall explain their relation later. One can obtain a junction condition for scalar field from its equation of motion. According to equation (3), we arrive at

$$\frac{\ddot{\phi}}{n^2} - \phi'' + \left(\frac{3\dot{a}}{an^2} - \frac{\dot{n}}{n^3} \right)\dot{\phi} - \left(\frac{n'}{n} + \frac{3a'}{a} \right)\phi' =$$
$$- V_{,\phi}(\phi) - \frac{2\beta}{M_p^{3/2}} \frac{1}{\sqrt{-h}} \frac{\partial \mathcal{L}_m}{\partial \tilde{h}_{\mu\nu}} \tilde{h}_{\mu\nu}\delta(y). \quad (25)$$

Matching the Dirac delta function in both side of this relation, for $y = 0$, we have

$$[\phi'] = \frac{2\beta}{M_p^{3/2}} \frac{1}{\sqrt{-h}} \frac{\partial \mathcal{L}_m}{\partial \tilde{h}_{\mu\nu}} \tilde{h}_{\mu\nu}. \quad (26)$$

The right hand side of this equation is computed as following

$$\frac{\beta}{M_p^{3/2}} \frac{1}{\sqrt{-h}} \frac{\partial \mathcal{L}_m}{\partial \tilde{h}_{\mu\nu}} \tilde{h}_{\mu\nu} = -\exp\left(\frac{4\beta\phi}{M_p^{3/2}} \right)$$
$$\times \frac{\beta}{M_p^{\frac{3}{2}}} \frac{-2}{\sqrt{-\tilde{h}}} \frac{\partial \mathcal{L}_m}{\partial \tilde{h}_{\mu\nu}} \tilde{h}_{\mu\nu}$$
$$= -\exp\left(\frac{4\beta\phi}{M_p^{3/2}} \right) \frac{\beta}{M_p^{3/2}} \left(\tilde{T}^{\mu\nu}\tilde{h}_{\mu\nu} \right)$$
$$= \frac{\beta(1 - 3\omega)}{M_p^{3/2}} \tilde{\rho}\exp\left(\frac{4\beta\phi}{M_p^{3/2}} \right), \quad (27)$$

where $\tilde{\rho}$ and $\tilde{p}$ are the energy density and pressure respectively in the A-frame. On the brane, the energy-momentum tensor in Einstein frame and A-frame are related to each other by

$$T^{\mu\nu} = \exp\left(\frac{6\beta\phi}{M_p^{3/2}}\right)\tilde{T}^{\mu\nu}, \quad (28)$$

therefore, ρ_b and p_b are easily expressed in term of the component of energy-momentum of A-frame and scalar

field as

$$\rho_b = \tilde{\rho}_0 \exp\left(\frac{(1-3\omega)\beta\phi}{M_p^{3/2}}\right)a_0^{-3(1+\omega)}, \qquad (29)$$

$$p_b = \omega\tilde{\rho}_0 \exp\left(\frac{(1-3\omega)\beta\phi}{M_p^{3/2}}\right)a_0^{-3(1+\omega)}, \qquad (30)$$

which $\tilde{\rho}_0$ is a constant that has been introduced in Eq.(7) and a_0 is scale factor that is taken on brane, namely $y = 0$. When scalar field approach to zero, there is $\rho_b = \tilde{\rho}$ and $p_b = \tilde{p}$. Also, because of the presence of Z_2-symmetry, as we have assumed before, one can attain a_0', n_0' and ϕ_0' function from the junction conditions. **From $(0,5)$ component of filed equation we have**

$$^5G_{05} = \kappa_5^2 T^{(\phi)05} = \kappa_5^2 \dot{\phi}\phi',$$

$(T^{(b)05} = 0)$. **Now, by substituting $\dot{a}'$ and n' form the junction conditions, (23), (24), and assuming Z_2-symmetry, one can obtained generalized continuity equation on the brane as**

$$\dot{\rho}_b + 3H(\rho_b + p_b) = 2\dot{\phi}_0\phi_0', \qquad (31)$$

as we expected the energy, due to the interaction between matter and scalar field, is not conserved. Note that in all of relation in this section we select $n_0 = 1$, without any less of generality. we recognize that, the generalized continuity equation explicitly confirms the junction condition. From $(5,5)$ component of field equation, one can obtain the second order (or generalized) Friedmann equation as

$$\frac{\ddot{a}_0}{a_0} + \frac{\dot{a}_0^2}{a_0^2} = -\frac{\kappa_5^4}{36}\rho_b(\rho_b+3p_b) - \frac{\kappa_5^2}{3}\left(\frac{\dot{\phi}_0^2}{2} + \frac{\phi_0'^2}{2} - V(\phi_0)\right). \quad (32)$$

The first Friedmann equation on the brane is obtained from $(0,0)$ component of field equation as

$$H^2 = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{\kappa_5^4}{36}\rho_b^2 + \frac{\kappa_5^2}{3}\left(\frac{\dot{\phi}_0^2}{2} + \frac{\phi_0'^2}{2} + V(\phi_0)\right) + \frac{\hat{a}_0''}{a_0}. \quad (33)$$

$\hat{a}_0''$ is the non-distributional part of double derivative of $a(t,y)$ with respect to the fifth coordinate, and the subscript 0 means that they are taken in $y = 0$. By ignoring $\frac{\hat{a}_0''}{a_0}$ and using $\rho_b = \rho + \sigma$, one can obtain

$$H^2 = \frac{\kappa_5^4\sigma}{18}\rho\left(1 + \frac{\rho}{2\sigma}\right) + \frac{\kappa_5^4}{36}\sigma^2 + \frac{\kappa_5^2}{3}\left(\frac{\dot{\phi}_0^2}{2} + \frac{\phi_0'^2}{2} + V(\phi_0)\right). \quad (34)$$

It is seen that equation(34) is agree with the results which is obtained in [8, 9] for brane world cosmology and is completely different with standard cosmology model, because in standard cosmology $H \propto \sqrt{\rho}$ rather than ρ.

III. THE EVOLUTION OF UNIVERSE

In this part we want to investigate the behavior of the evolution of universe in early and late times. One can

rewrite the generalized Friedmann equation as following

$$\frac{\ddot{a}_0}{a_0} = -\frac{\kappa_5^4}{36}(2+3\omega)\rho_b^2 - \frac{\kappa_5^2}{3}(\dot{\phi}_0^2 + \phi_0'^2). \qquad (35)$$

From the junction condition, and with the help of the function of energy density, we rearrange this relation as

$$\frac{\ddot{a}_0}{a_0} = -\frac{\kappa_5^4}{36}(2+3\omega)\left(\frac{\tilde{\rho}_0}{a_0^{3(1+\omega)}}\right)^2$$
$$\times \exp\left(\frac{2(1-3\omega)\beta\phi}{M_p^{3/2}}\right) - \frac{\kappa_5^2}{3}\dot{\phi}^2$$
$$-\frac{\kappa_5^2}{3}\frac{\beta^2}{4M_p^3}(1-3\omega)^2\left(\frac{\tilde{\rho}_0^2}{a_0^{6(1+\omega)}}\right)^2$$
$$\times \exp\left(\frac{2(1-3\omega)\beta\phi}{M_p^{3/2}}\right) \qquad (36)$$

A. Early time

In early times, namely in inflation stage, the scalar field is dominant and also it is well known that the scalar potential energy of the inflation dominates over the kinetic energy. So that the equation (35) reduces to

$$\frac{\ddot{a}}{a} \simeq -\frac{\kappa_5^4}{3}\phi_0'^2,$$
$$= -\frac{\kappa_5^4}{3}\frac{\beta^2(1-3\omega)^2}{M_p^3}\frac{\tilde{\rho}_0^2}{a^{6(1+\omega)}}\exp\left\{\frac{8\beta\phi_0}{M_p^{3/2}}\right\}. \quad (37)$$

The effect of $[\exp\{4\beta\phi_0/m_p^{3/2}\}/a^{3(1+\omega)}]^2$ on evolution of universe is obvious in this area. This term is large and may explain inflation in very early step of universe evolution. At the end of inflation the evolution of the universe has a transition from a de Sitter stage, during which the evolution of the universe is dominated by the scalar field, to a subsequent radiation or matter dominated Friedmann-Robertson-Walker type cosmological model. One of the possible approaches to this problem is phenomenological [23, 26, 27]. At this stage the first term of (35) is dominated and then we have

$$\frac{\ddot{a}}{a} \simeq -\frac{\kappa_5^4}{36}(2+3\omega)\frac{\tilde{\rho}_0^2}{a_0^{6(1+\omega)}}\exp\left(\frac{2(1-3\omega)\beta\phi}{M_p^{3/2}}\right). \quad (38)$$

In radiation dominant with $\omega = 1/3$, the power of exponential function vanishes and we obtain a deceleration phase of expansion for universe. However, in this stage we want to investigate the inflation and then reheating process of the evolution after inflation.

1. Inflation

In the widely accepted inflationary scenario it is assumed that during an initial period the universe

is dominated by a large, approximately constant potential term $V(\phi)$ of a scalar field ϕ_0, known as the inflaton field [22, 23]. In our model the energy-momentum tensor of the scalar field can be written on the brane in the perfect fluid form, with energy density ρ_ϕ and pressure p_ϕ as

$$\rho_{\phi_0} = \frac{1}{2}\dot{\phi}_0^2 + \tilde{V}_{eff}(\phi_0), \tag{39}$$

$$p_\phi = \frac{1}{2}\dot{\phi}_0^2 - \tilde{V}_{eff}(\phi_0), \tag{40}$$

where

$$\tilde{V}_{eff}(\phi_0) = V(\phi_0) + \frac{1}{2}\phi_0'^2 \tag{41}$$

and $\phi_0 = \phi(y,t)|_{y=0} = \phi_0(t)$. So that in this case equation (34) reduces to

$$H^2 = \frac{\kappa_5^4}{36}\sigma^2 + \frac{\kappa_5^2}{3}\rho_{\phi_0}. \tag{42}$$

Let us now consider the equation of motion of scalar field on the brane. Eq.(3) can be rewritten on the brane as

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \frac{\kappa_5^2}{3}(\rho_b - 3p_b)\phi_0' = -V_{,\phi}(\phi_0), \tag{43}$$

we have ignored the non distributional part of ϕ''. This equation clearly show the effect of bulk on the equation of motion of ϕ. We may introduce an effective potential to describe the dynamic of scalar field as

$$V_{eff,\phi}(\phi_0) \equiv V_{,\phi}(\phi_0) + \frac{\kappa_5^2}{3}\left[\{1 - 3\omega\}\rho_b\phi_0'\right]. \tag{44}$$

Dependence on energy density of effective potential and some quantity which have come form bulk is explicit. Hence, one can easily write the equation of motion for scalar field on the brane as

$$D^2\phi = -V_{eff,\phi}(\phi_0), \tag{45}$$

where D^2 is the D'Alembert of scalar field in four-dimensional space-time. It is as same as the equation which is obtained for low energy, 4D [5, 6]. So that one can obtain the mass of the scalar field ϕ as

$$\begin{aligned} m_\phi^2 &= V_{eff,\phi\phi}, \\ &= V_{,\phi\phi} + \frac{\kappa_5^2}{3}\{1 - 3\omega\}\frac{\partial}{\partial\phi}(\rho_b\phi_0'). \end{aligned} \tag{46}$$

Therefore, although bulk is free of matter, but due to interaction between the matter and scalar field, scalar field which propagate through the bulk takes a mass which is depending to the brane density energy. In fact this means that the mass of scalar field in the 4D effective action is not

small and then the correction to the Newton law can not be large by the propagation of the scalar field in the bulk. We want to obtain a simple form for m_ϕ^2. Using (24) and (25) we have

$$\begin{aligned} \phi_0' &= \frac{1}{2}[\phi_0'], \\ &= \frac{\beta(1 - 3\omega)}{M_p^{3/2}}\frac{\tilde{\rho}_0}{a^{3(1+\omega)}}\exp\left\{\frac{4\beta\phi_0}{M_p^{3/2}}\right\}. \end{aligned} \tag{47}$$

Using (29), (30) and (47) we have

$$\frac{\kappa_5^2}{3}(\rho_b - 3p_b)\phi_0' = A(t)\exp\left\{\frac{\beta(5 - 3\omega)\phi_0}{M_p^{3/2}}\right\}, \tag{48}$$

where

$$A(t) = \frac{\beta\kappa_5^2\tilde{\rho}_0^2(1 - 3\omega)^2}{3a^{6(1+\omega)}M_p^{3/2}}.$$

Substituting (48) in (44) and (46) give

$$V_{eff,\phi} = V_{,\phi}(\phi) + A(t)\exp\left\{\frac{4\beta\phi_0}{M_p^{3/2}}\right\}, \tag{49}$$

$$m_\phi^2 = V_{,\phi\phi} + \frac{4\beta}{M_p^{3/2}}A(t)\exp\left\{\frac{4\beta\phi_0}{M_p^{3/2}}\right\}. \tag{50}$$

It is well known that the potential energy of the inflation dominates over the kinetic energy this means $V_{eff}(\phi_0) \gg \dot{\phi}_0^2/2$. Hence one requires a flat potential for the inflation in order to lead to sufficient inflation. Implying the slow-roll conditions: $V_{eff}(\phi_0) \gg \dot{\phi}_0^2/2$ and $|\ddot{\phi}_0| \ll 3H|\dot{\phi}_0|$ Eqs. (42) and (43) are approximately given as

$$H^2 = \frac{\kappa_5^2}{3}V_{eff}(\phi_0), \tag{51}$$

$$3H\dot{\phi}_0 = -V_{eff}'(\phi_0), \tag{52}$$

the velocity is negative for $V_{eef} > 0$, because the filed rolls down the potential towards smaller ϕ_0 values.

2. Reheating after Inflation

During a second period of evolution, the potential minimum is approached, $V_{eff}(\phi_0)$ tends to zero, and the scalar field starts to fluctuate violently around the minimum value. In fact at the end of inflation some of scalar field energy density needs to be converted to conventional matter to restore hot big bang cosmology usually this process comes by decay of scalar field (inflaton field). In addition there is the possibility of reheating by gravitational particle production, where the required particles are produced quantum mechanically from time varying gravitational field. The method of studying for this kind of reheating depends on scalar field equation of motion

and expansion rate, namely Eq.(43) and (42) [35]. However, in the post-inflationary stage, the inflaton field executes coherent oscillations about the minimum of the potential [39] and the kinetic term dominates the potential term in the reheating era. Therefore we can expand the effective potential around the minimum point and in reheating area we have

$$V_{eff}(\varphi) \simeq \frac{1}{2}m_{\phi_{min}}^2\varphi^2, \qquad (53)$$

where we have ignored a constant in (53), $\varphi = \phi_0(t) - \phi_{0_{min}}$ and the effective potential is minimized at $\phi_{0_{min}}$. Then the scalar field equation of motion in reheating area is as

$$\ddot{\varphi} + 3H\dot{\varphi} + m_{\phi_{min}}^2\varphi = 0, \qquad (54)$$

According to the theory of reheating, [36, 37], was based on the concept of single-body decays, the inflaton field is a collection of scalar particles each with a finite probability of decaying. Such decays can be treated by coupling ϕ, to other scalar or fermion fields through terms in the Lagrangian. We assume the inflaton field is coupled with a matter scalar filed as $K\varphi\psi^2$. Here ψ is matter scalar field. Since the homogeneous part of the inflaton is very large at the end of inflation it behaves like a classical field. So that inflaton treat as a classical external force and acting on the quantum fields ψ. The explicit expression of the decay width of the scalar field can be represented as [38]

$$\Gamma = \alpha_\phi m_\phi \sqrt{1 - (\frac{T}{m_\phi})^2} \qquad (55)$$

where α_ϕ and m_ϕ are the coupling constant and the mass of inflation respectively. T is the decaying temperature and it can be related to the matter density, ρ_m as

$$\rho_m = \sigma T^{\frac{\gamma}{\gamma-1}}, \qquad (56)$$

where σ is a constant. For radiation dominant, $\rho_m = \pi^2 T^4/15$, $\sigma = \pi^2/15$, and for matter-dominated universe, the relation between ρ_m and T can also be written down explicitly. The scalar field is negligible small in the matter-dominated phase and in the non-relativistic matter domination era $T \leq 1\text{eV}$. This is far smaller than the minimum bound obtained for m_ϕ especially in our model which the scalar field is coupled to mater and then the mass of scalar field is depend on local matter. Hence, in the non-relativistic phases of matter evolution the decay rate is simply$\Gamma = \alpha_\phi m_\phi$.
In order to obtain abetter insight, we need to know the numerical values of the model parameters. Follow to the above mention the order of magnitude of $\alpha_\phi m_\phi$ is about the order of magnitude of the decay width, which is the reciprocal of the characteristic timescale of reheating. The inflationary era ends, and reheating can start at the earliest at around $t = 10^{-32}$ s, while the hot big

bang commences at around $t = 10^{-18}$ s [24, 25]. The reheating process should complete before the hot big bang to restore the BBN. Therefore $10^{18}\text{s}^{-1} \leq \Gamma \leq 10^{32}\text{s}^{-1}$ and this require that

$$1\text{KeV} \leq \alpha_\phi m_\phi \leq 10^8\text{GeV}. \qquad (57)$$

Note that in our model m_ϕ^2 is time dependent and this fact was one of the main insight of inflationary and reheating cosmology in 1990 decade. For explaining the preheating we must couple the inflaton flied φ to another matter scalar field through an interaction term in the Lagrangian (1) as

$$\mathcal{L}_{int} = (1/2)g^2\phi^2\psi^2, \qquad (58)$$

where g is a dimensionless coupling constant. The total effective potential for this system will be the sum of the effective potential, V_{eff} driving inflation which was independent of ψ and the above interaction term:

$$U_{eff} = V_{eff}(\phi) + (1/2)g^2\varphi^2\psi^2. \qquad (59)$$

Note that the infaton field has two interaction with matter in this model. One of the is created with chameleon mechanism and the other one is created as (58). According to the above equation, ψ, with zero bare mass, will find an effective mass as

$$m_\psi = g\phi(t). \qquad (60)$$

It is well known, in our model, m_ψ is very smaller then m_ϕ and then according to [87, 237] this plying the crucial role in the reheating process. So one can obtain the Fourier modes of the ψ field which obey the following relation

$$\ddot{\psi}_k + 3H\dot{\psi}_k = -[\frac{k^2}{a^2} + g^2\phi^2(t)]\psi_k, \qquad (61)$$

Using $\psi = a^{-3/2}X(t)$, where $a(t)$ is the scale of FLRW universe, we have

$$\ddot{X}(t) - \omega_k^2 X(t) = 0, \qquad (62)$$

where

$$\omega_k = \sqrt{\frac{k^2}{a^2} - \frac{3}{2}\dot{H} - \frac{9}{4}H^2 + g^2\phi(t)^2}. \qquad (63)$$

It is seen that this equation is an oscillating equation with time dependence frequency ω_k and the crucial parameter in this ω is m_X^2. If this quantity, m_X^2 is changing rapidly then ω_k change also. This is quantified by the dimensionless parameter R_a which is defined as

$$R_a \equiv \frac{\dot{\omega}_k}{\omega_k^2}. \qquad (64)$$

The $R_a \ll 1$ is usually known as the adiabatic regime. In this regime there is no particle creation and then the number of particles are constant. But if $R_a \gg 1$ the

number of particles are not constant, this means that in this regime of model there is particle creation during the reheating. One can obtain the adiabaticity parameter as

$$\frac{|\dot{\omega}_k|}{\omega_k^2|} \sim \frac{|\dot{\varphi}|}{g\varphi^2}, \qquad (65)$$

in the interval $|\Delta\varphi| \leq \sqrt{|\dot{\varphi}|/g}$ the adiabaticity parameter, equation (65), is greater than $\mathcal{O}(1)$. Here $\dot{\varphi}$ is evaluated at the collision time.

Obtaining an explicit relation for reheating temperature, e-folding number and other relevant quantity the the reheating process need further study and investigation for some typical example. So this still is an open problem in our model.

B. Late time

The present model is a slow rolling model. The Percentage change in the expectation value of ϕ in one Hubble time, $\tau = \dot{\phi}/\phi H$ is a criterion for the slow rolling limit of a scalar field. If this quantity be $\tau \approx V_{eff,\phi}/\phi H$; so, if this ratio is sufficiently small, the scalar field is in the slow rolling limit. In this case $\dot{\phi}$ is very small and we can illegal $\dot{\phi}^2$. Therefore we arrive at

$$\frac{\ddot{a}}{a} \simeq -\frac{\kappa_5^2}{3}\left(\frac{1}{2M_5^3}(2+3\omega) + \frac{\beta^2}{4M_p^3}(1-3\omega)^2\right)$$
$$\times \left(\frac{\tilde{\rho}_0}{a_0^{3(1+\omega)}}\right)^2 \exp\left(\frac{2(1-3\omega)\beta\phi}{M_p^{3/2}}\right). \qquad (66)$$

If we take the magnitude of M_5 in order of electroweak scale $M_E = 1TeV$ [16] and M_p in order of $10^{18}GeV$, the second term in parenthesis may be ignored with respect to the first term. So, to have an accelerated expanding universe in this area ω should obey as following

$$\omega < -\frac{2}{3}, \qquad (67)$$

this range for ω is consistent with astronomical data, because the most recent data indicate that $\omega < -0.76$ at the 95% confidence level[17]. This consistency shows that in this epoch the two models (standard cosmology and brane world cosmology with chameleon scalar field in the bulk) have the same observational results, because in this epoch(late time) one can ignore ρ^2 term in equation (35) and then this model can be reduces to the standard cosmology model. For $-1 < \omega < -\frac{2}{3}$, with attention to the power of a_0, there is a positive and decreasing value for $\ddot{a}$ and for $\omega < -1$ there is a positive and increasing value.

IV. VALIDITY OF GENERALIZED SECOND LAW OF THERMODYNAMIC

One of important question concern the thermodynamics behavior of universe. The first connection between general relativity and thermodynamics was given by Bekenstein in 1973. Since then , thermodynamics aspect of universe has been the subject of several studies and validity of thermodynamics law have been investigated is many works. In recent year, there are alot of interest in the brane world scenario which gave us a new interesting picture of universe and a generalized form of universe. Maybe the main reason for studying the validity of thermodynamics law is that it is natural to study this subject for models which have been bielt is this new scenario.

Let us now, examine the validity of generalized second law of thermodynamic (GSLT), with assumption of validity the first law. We consider a region of FLRW universe involved by horizon. There is some works for validity of GSLT in DGP brane world scenario [18, 19]. This bounded region is filled with a perfect fluid, where

$$p_b = \omega\rho_b.$$

Amount of energy acrossing the horizon in time dt is equal to

$$\frac{dE}{dt} = \frac{4\pi}{3}R_h^3\left(-3H(\rho_b + p_b) + 2\phi'\dot{\phi}\right), \qquad (68)$$

where R_h is the radius of the horizon. With the help of validity of the first law of thermodynamic, variation of horizon entropy is expressed as

$$\dot{S}_h = \frac{4\pi R_h^3}{3T_h}\left(3H(\rho_b + p_b) - 2\phi'\dot{\phi}\right) \qquad (69)$$

T_h is the temperature of horizon. Using the Gibbs's equation,

$$T_h dS_I = dE_I + p_b dV.$$

Note that according to [20], we have supposed a equilibrium for temperature of inside matter and horizon. Time evolution of the entropy of inside horizon is obtained as

$$\dot{S}_I = \frac{1}{T_h}\left(V\dot{\rho}_b + (\rho_b + p_b)\dot{V}\right). \qquad (70)$$

Since we have $V = \frac{4\pi}{3}R_h^3$ and $E = \rho_b V$, so $\dot{S}_I$ can be rewritten as

$$\dot{S}_I = \frac{4\pi R_I^2}{T_h}$$
$$\times \left(\frac{R_I}{3}(-3H(\rho_b + p_b) + 2\phi'\dot{\phi}) + (\rho_b + p_b)\dot{R}_I\right) \quad (71)$$

Adding Eqs.(69) and (71), we attain the total variation of entropy

$$\dot{S} = \dot{S}_h + \dot{S}_I = \frac{4\pi R_h^2}{T_h}(\rho_b + p_b)\dot{R}_I \qquad (72)$$

Now, we are ready to examine the validity of GSLT for apparent horizon and cosmological event horizon. In the following we compute the time variation of R for both of this horizon.

- **Apparent Horizon:**
  For flat space of our universe, apparent horizon is described by $R_A = \frac{1}{H}$. So we have

$$\dot{R}_A = -\frac{\dot{H}}{H^2}. \tag{73}$$

The sign of $\dot{R}_A$ depends on the sign of $\dot{H}$.

- **Cosmological Event Horizon:**
  The radius of the cosmological event horizon is given by

$$R_E = a_0(t) \int_t^\infty \frac{dt}{a_0(t)}.$$

So the time variation of R_E is described by

$$\dot{R}_E = HR_E - 1. \tag{74}$$

According to [21], for a universe with a positive acceleration, like our model, If $\dot{H} > 0$ so, there is

$$\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} > 0 \tag{75}$$

by integration above relation

$$\int_t^\infty \frac{d\dot{a}}{\dot{a}^2} > \int_t^\infty \frac{dt}{a}. \tag{76}$$

For an accelerate expanding universe, $\ddot{a} > 0$, in the late time $\dot{a}(t \longrightarrow \infty)$ goes to infinite. We can compute the above integral as

$$\dot{a} \int_t^\infty \frac{dt}{a} < 1, \tag{77}$$

it means $HR_E < 1$. Therefore there is $\dot{R}_E < 0$ for $\dot{H} > 0$ and $\dot{R}_E > 0$ for $\dot{H} < 0$.

With attention to the above result, one see that for investigation the validity of GSLT we need to determine the sign of $\dot{H}$. we carry out it with the help of first Friedmann equation. By taking time derivative of the first Friedmann equation, we arrive at

$$2H\dot{H} = -\frac{\kappa_5^4}{6}H(1+\omega)\rho_b^2 + \frac{\kappa_5^2}{9}\rho_b\phi'\dot{\phi}$$
$$+\frac{\kappa_5^2}{3}\dot{\phi}(\ddot{\phi} + V_{,\phi}(\phi)) + \frac{\kappa_5^2}{3}\phi'\dot{\phi}'. \tag{78}$$

The reader should note some points: first, we have an acceleration expanding universe, so H is always positive; second, if we suppose that the scalar field be a uniform decreasing function of time, then the time derivative of it is negative; third, $V_{,\phi}(\phi)$, as we can see in [5, 6], is always negative, so $\dot{\phi}V_{,\phi} > 0$. Substituting the energy

density and with the help of junction condition, one can rearrange the above equation

$$2H\dot{H} = -\frac{\kappa_5^2}{3}\tilde{\rho}_0^2 \exp\left(\frac{2(1-3\omega)\beta\phi}{\sqrt{M_p^3}}\right)a^{-6(1+\omega)}$$
$$\left\{\kappa_5^2\left(-\frac{H(1+\omega)}{2} + \frac{\beta}{3\sqrt{M_p^3}}(1-3\omega)\dot{\phi}\right)\right.$$
$$\left.+\frac{\beta^2(1-3\omega)^2}{M_p^3}\left(\frac{(1-3\omega)\beta\phi}{\sqrt{M_p^3}} - H\right)\right\}$$
$$+\frac{\kappa_5^2}{3}\dot{\phi}\left(\ddot{\phi} + V_{,\phi}(\phi)\right) \tag{79}$$

Since the $\dot{\phi}$ is small and because of the existence of the M_p in the denominator we can estimate

$$2H\dot{H} \longrightarrow -\frac{\kappa_5^2}{6}H(1+\omega)\tilde{\rho}_0^2 \exp\left(\frac{2(1-3\omega)\beta\phi}{\sqrt{M_p^3}}\right)a^{-6(1+\omega)}$$
$$+\frac{\kappa_5^2}{3}\dot{\phi}V_{,\phi}(\phi) \tag{80}$$

to have $\dot{H} > 0$, especially in early times, there should be $\omega < -1$. This result indicate that $\dot{H} > 0$, so the time variation of both apparent horizon and cosmological event horizon are negative, namely $\dot{R}_E, \dot{R}_A < 0$. For validity the GLST the equation of state parameter should be smaller than -1, this is compatible with relation (26). In this range of ω our matter is phantom. If $-1 < \omega < -\frac{2}{3}$, there is still an accelerated expansion, and for validity of GSLT, $\dot{H}$ should be negative. It means the first term on the right hand side of (44) should be dominant. With considering equation (44) we see that this condition is not unreasonable. In this range value of ω our matter is named quintessence.

V. CONCLUSION

In this paper a kind of scalar field is embedded in the bulk space-time, called chameleon scalar field. **A brief review on gravity localization has been mentioned, where by taking some gauge as well as transverse-traceless gauge, a wave equation for the purturbation of the metric could be achieved to let us determine whether or not gravity localized on brane.** The junction conditions is acquired. The junction condition for component of metric is similar the result of the work of Binetruy *et al*, but in contrast to their result, here the components of energy-momentum tensor depend on the scalar field with a exponential term. Considering the generalized Friedmann equation tells us there is an exponential expansion in early times. This exponential expansion may have relation with inflation period. The assumption of interaction between matter and

scalar field bring us an effective potential to recover basic equation for explaining inflation. It has been shown that in radiation dominant era, the exponential term is omitted from acceleration equation. Also we have a positive acceleration in late time for universe. The equation of motion of scalar field on the brane is similar to the standard model and it displays that the mass of scalar field depends on the local energy density. Investigation the validity of generalized second law of thermodynamic for apparent horizon and cosmological event horizon show that the matter on the brane can be phantom or quintessence.

VI. AKNOWLEDGEMENT

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